Estimation of two layer soil parameters using traditional and evolutionary methods

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Abstract— In this paper, the optimum values of the parameters of a two-layer soil model, are estimated by a traditional method and an evolutionary method.

Both the methods start with the well-known Wenner resistivity measurement data. In Wenner method of measuring soil resistivity, a set of apparent resistivity measurements are made with different electrode spacing. For each measured value of apparent resistivity, the same is computed in terms of soil parameters, which are the upper layer soil resistivity, the lower layer soil resistivity and the height of the upper layer. Then optimal values of soil parameters are iteratively estimated. Optimal values are those values of soil parameters for which the sum of absolute values of normalised error is the minimum. The difference between measured and computed values of soil resistivity is termed as error.

Particle Swarm Optimization is used as the evolutionary method in this paper. The search process is accelerated by a search space reduction technique to obtain the results in minimum number of iterations and time.

The algorithms have been tested on nine sets of test data and the results obtained are better than all the results published so far.

***Keywords*— Particle Swarm Optimization, Search space reduction, Soil parameter estimation, Soil resistivity, Wenner method, Evolutionary Computation, Apparent resistivity.**

1. INTRODUCTION

The Wenner four probe method of measuring soil resistivity is very popular among ground mat design engineers. Soil resistivity is rarely uniform and a two-layer model is widely accepted as the best for ground mat design purpose. In Wenner method, four probes are used for soil resistivity measurement. They are placed along a straight line and driven into earth with equal spacing among them. The depth of penetration of a probe into earth is adjusted approximately to ten percent of the probe spacing. The outer probes are meant for passing a current through earth and the inner probes are meant for measuring the potential difference between them.

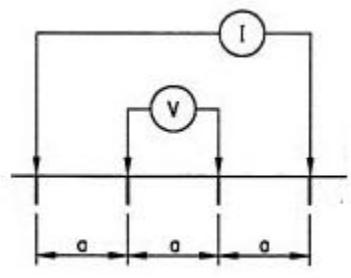

Fig.1 Wenner four probe method of measuring apparent resistivity

As soil resistivity is found to vary with probe spacing, a set of measurements are made with different electrode spacing for estimating the parameters of the two-layer soil model. Using the measurement data, apparent resistivity corresponding to each measured value are computed using (1).

$$\rho_a = \rho_1\left[1+4\sum_{n=1}^{\infty}\left[\left(\frac{k^n}{\sqrt{1+(2nh/a)^2}}\right)-\left(\frac{k^n}{\sqrt{4+(2nh/a)^2}}\right)\right]\right] \quad (1)$$

In (1)
ρ_1 is the upper layer soil resistivity,
k is the reflection factor given by $(\rho_2-\rho_1)/(\rho_2+\rho_1)$,
ρ_2 is the lower layer soil resistivity,
h is the height of the upper layer and
a is the probe spacing.
ρ_1, ρ_2 and h are the three parameters of the two-layer soil model.

The infinite term given in (1) is computed a finite number of times depending on the accuracy requirement; 2000 is a quite reasonable value for the upper limit of 'n' as suggested by researchers.

If a set of 'm' measurements are made, each measurement corresponding to particular electrode spacing, then

apparent resistivity values are computed for each measured value. As computed value of apparent resistivity is a function of ϱ_1, ϱ_2 and h, the optimization problem is stated as locating values for ϱ_1, ϱ_2 and h such that the objective function F is minimum and stated as (2).

$$minimize\ F = \sum_{i=1}^{m}((abs(\rho_{ai}^{meas} - \rho_{ai}^{comp}))/\rho_{ai}^{meas}) \quad (2)$$

here ρ_{ai}^{meas} is the i^{th} measured value of apparent resistivity and ρ_{ai}^{comp} is the i^{th} computed value of apparent resistivity.

Research has been going on in this filed for decades together and plenty of literature is already available on estimation of two-layer soil parameters. Additional information on determination of soil parameters using traditional and evolutionary methods can be obtained from [1] - [7].

2. TRADITIONAL ALGORITHM

The traditional method used here is an unconstrained, non-linear optimization algorithm in which F, as suggested by (2), is minimized without subjecting it to any constraint. F is zero when measured and computed values of apparent resistivity are the same in all the 'm' cases. As apparent resistivity values are to be computed in 'm' cases, there is a set of 'm' non-linear equations which are split into a set of 'm' linear equations and a correction part. The equations are solved iteratively to obtain the optimum values of soil parameters.

The linear equations are of the form *A=B*C* where

A is the vector of difference between measured and computed values of apparent resistivity,

B is the matrix of partial derivatives of ϱ_a given in (1) with respect to ϱ_1, ϱ_2 and h

C is the correction vector consisting of elements $\Delta\varrho_1$, $\Delta\varrho_2$ and Δh.
A is of size m x1, B is of size m x 3 and C is of size 3 x 1.

When the set of linear equations are solved, the correction vector C can be found using (3).

$$C = (B^T I\ B)^{-1}(B^T I\ A)\ \alpha \quad (3)$$

B^T is the transpose of matrix B and I is an identity matrix of size m x m. α is a factor which decides the rate of convergence. The choice of α is normally by trial and error. But sufficient care should be taken as a wrong choice of α results in failure of convergence.

Now the elements of A and B are to be determined. If there are 'm' measurements, then for each measured value of apparent resistivity, the same is computed using (1). The difference between measured and computed values is found in all the 'm' cases to form vector A.

For determining the elements of B, the equations given Fig. 2 are made use of which give the partial derivatives of ϱ_a with respect to ϱ_1,ϱ_2 and h. To evaluate these equations, an initial set of values of ϱ_1,ϱ_2 and h are essential. These values can be suitably assumed. A better practice is to take the initial guess value of ϱ_1 as the measured value of apparent resistivity with minimum electrode spacing and the initial guess value of ϱ_2 as the measured value of apparent resistivity with maximum electrode spacing. Initial guess value of 'h' is taken as half of the maximum electrode spacing.

$$\frac{\partial\rho_a}{\partial\rho_1} = 1 + 4\sum_{n=1}^{\infty}\left[\left(1 - \frac{n(1-k^2)}{2k}\right)\left(\frac{k^n}{\sqrt{1+(2nh/a)^2}} - \frac{k^n}{\sqrt{4+(2nh/a)^2}}\right)\right]$$

$$\frac{\partial\rho_a}{\partial\rho_2} = \sum_{n=1}^{\infty}\left[\left(\frac{2n}{k}(1-k^2)\right)\left(\frac{k^n}{\sqrt{1+(2nh/a)^2}} - \frac{k^n}{\sqrt{4+(2nh/a)^2}}\right)\right]$$

$$\frac{\partial\rho_a}{\partial h} = \frac{16\rho_1 h}{a^2}\sum_{n=1}^{\infty}\left[\left(\frac{k^n}{\left(4+(2nh/a)^2\right)^{1.5}}\right) - \left(\frac{k^n}{\left(1+(2nh/a)^2\right)^{1.5}}\right)\right]$$

Fig.2 Partial derivates of ϱ_a with respect to ϱ_1, ϱ_2 and h

2.1 Structure of A,B and C

An element of vector A is the difference between the measured and computed values of apparent resistivity. Vector A is of size m x1. Matrix B contains elements which are the partial derivatives of ϱ_a with respect to ϱ_1, ϱ_2 and h. There are 'm' rows, where each row corresponds to a measurement. The matrix is of size m x 3. Vector C has correction elements to update the values of ϱ_1, ϱ_2 and h in iteration. It is of size 3x1. The structures are shown in Fig.3.

$$A = \begin{bmatrix} \rho_{a1}^{meas} - \rho_{a1}^{comp} \\ \rho_{a2}^{meas} - \rho_{a2}^{comp} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \rho_{am}^{meas} - \rho_{am}^{comp} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial \rho_{a1}}{\partial \rho_1} & \frac{\partial \rho_{a1}}{\partial \rho_2} & \frac{\partial \rho_{a1}}{\partial h} \\ \frac{\partial \rho_{a2}}{\partial \rho_1} & \frac{\partial \rho_{a2}}{\partial \rho_2} & \frac{\partial \rho_{a2}}{\partial h} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \frac{\partial \rho_{am}}{\partial \rho_1} & \frac{\partial \rho_{am}}{\partial \rho_2} & \frac{\partial \rho_{am}}{\partial h} \end{bmatrix}$$

$$C = \begin{bmatrix} \Delta\rho 1 \\ \Delta\rho 2 \\ \Delta h \end{bmatrix}$$

Fig.3. Structure of A.B and C

2.2 Step-by-step procedure for estimating optimal values of soil parameters by traditional algorithm

1. Assign suitable initial values for ϱ_1, ϱ_2 and h. Start iteration count. Specify tolerance.
2. Evaluate elements of A using the most recent values of ϱ_1, ϱ_2 and h.
3. Evaluate elements of B using the equations given in Fig.2 with the most recent values of ϱ_1, ϱ_2 and h.
4. Compute $C = (B^T I\ B)^{-1}(B^T I\ A)\ \alpha$
5. If $\Delta\varrho_1$, $\Delta\varrho_2$ and Δh, the elements of C, are less than the specified tolerance, go to step 7.
6. Modify the present values of ϱ_1, ϱ_2 and h to $(\varrho_1+\Delta\varrho_1)$, $(\varrho_2+\Delta\varrho_2)$ and $(h+\Delta h)$. Advance iteration count and go to step 2.
7. Display the final values of ϱ_1, ϱ_2 and h.
8. Stop

3. THE PSO ALGORITHM

The PSO method eliminates most of the mathematical computations involved in the traditional algorithm. The basic PSO algorithm for obtaining the optimal values of soil parameters ϱ_1, ϱ_2 and h is the following.

A population of 'L' number of trial values is generated for each soil parameter. With one trial value for a parameter from each population, apparent resistivity values are computed for 'm' measurements and the objective function stated in (2) is evaluated. The process is repeated for the next set of trial values and so on till the end of population. For each set of trial values personal best is preserved and at the end of population global best is identified. Now the population elements are modified and the process is repeated with the modified population elements. The objective function value keeps reducing and the process is terminated after a definite number of iterations.

3.1 Search Space Reduction

The normal PSO algorithm explores a large space for locating optimal values of ϱ_1, ϱ_2 and h. The search space can be reduced and the process can be expedited using the following technique.

In any traditional iterative method, initial guess values of the soil parameters ϱ_1, ϱ_2 and h are mandatory for solving the problem; also these guess values should be close to the final converged values. This aspect can be incorporated in the PSO algorithm to reduce the search space. Thus the initial search space for each soil parameter is defined by covering a range above and below its initial guess value. For example, at start, the upper limit of a soil parameter can be set as thrice the initial guess value and the lower limit as zero. The initial guess value is found in the traditional algorithm.

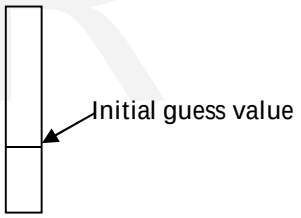

Fig. 4. Initial search space for a soil parameter

Hence to reduce the search space, the unconstrained optimization problem is converted to a constrained problem by imposing limits on variation of the values of soil parameters ϱ_1, ϱ_2 and h. The modified optimization problem is stated as (4).

$$minimize\ F = \sum_{i=1}^{m}((abs(\rho_{ai}^{meas} - \rho_{ai}^{comp}))/\rho_{ai}^{meas})$$

subject to the imposed constraints

$$\rho_1^{min} \le \rho_1 \le \rho_1^{max}$$

$$\rho_2^{min} \le \rho_2 \le \rho_2^{max}$$

$$h^{min} \le h \le h^{max} \qquad (4)$$

The objective function mentioned above is constrained by upper and lower limits of soil parameters ϱ_1, ϱ_2 and h at any

point of time during the iteration process. Initial guess values are found as in the traditional algorithm. Now the upper and lower limits of ϱ_1, ϱ_2 and h can be fixed. The upper limit of a parameter can be 'k_1' times the initial guess value and the lower limit can be zero. Normally k1 is assigned a value greater than 2. Due to the imposed constraints, the search for a parameter is carried out only in the range defined.

As the search progresses, the upper and lower limits of soil parameters are reset about their most recent global best values instead of initial guess values. The upper limits of the parameters are set as 'k_1' times the most recent global best value of that parameter and the lower limits are set as (2-k_1) times the global best value. The limits are fixed as given in (4) with starting value for k_1 equal to 2. This means that the upper limit of the search space of a soil parameter has become twice its most recent global best value with lower limit remaining zero. Subsequently, say, when k_1 is reduced to 1.5 on improvement in global best fitness value, the upper limit and lower limit of a parameter become 1.5 times and 0.5 times its most recent global best value. The value of k_1 can be gradually reduced to 1.1. This means that, at any point of time during the search, we are trying to locate the optimal value of a parameter in a narrow range above and below its most recent global best value. Fig.5 shows the reduction in search space as iteration progresses.

$$\rho_1^{min} = (2-k_1) * \rho_{1gbest}, \quad \rho_1^{max} = k_1 * \rho_{1gbest}$$
$$\rho_2^{min} = (2-k_1) * \rho_{2gbest}, \quad \rho_2^{max} = k_1 * \rho_{2gbest}$$
$$h^{min} = (2-k_1) * h_{gbest}, \quad h^{max} = k_1 * h_{gbest} \qquad (5)$$

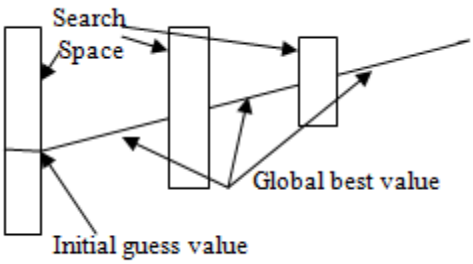

Fig. 5. Reduction in search space

Fitness = 1/ (1+F). (6)

3.2 Step-by-step procedure for estimating optimal values of soil parameters

1. Initialize a matrix A of size L x 4, the first three columns of which are for retaining trial values of ϱ_1, ϱ_2 and h and the fourth column, to retain computed fitness values. The elements of the first three columns are initialized with random numbers mapped to the initial search range using standard mapping rule.
2. Copy the elements of A to another matrix B. In due course, B will retain the best personal values of ϱ_1, ϱ_2, h and the corresponding fitness value.
3. Consider the set of trial values for ϱ_1, ϱ_2 and h from the first row of A.
4. Compute the apparent resistivity for each measured value of resistivity using (1).
5. Compute fitness using (6) and store the value in A as the fourth column element of the same row.
6. Compare the fitness values in A and B. If the fitness value in A is higher, copy the row elements to the corresponding positions in B.
7. Extract the set of trial values of ϱ_1, ϱ_2 and h from the next row of A.
8. If not end-of-matrix, go to step 4.
9. Locate the highest fitness value in B and the corresponding global best values of ϱ_1, ϱ_2 and h. If fitness is more than 0.999 or iteration count exceeds 500, stop the process.
10. Modify the search space using (5) by reducing the value of k_1 depending on the current best global fitness value. Modify all the values of ϱ_1, ϱ_2 and h in matrix A without violating limits. Go to step 3.

3.3 Modifying the values of ρ_1, ρ_2 and h

For modifying the trial values of ϱ_1, ϱ_2 and h in matrix A, the following well known equation is used.

$$v^{modified} = w*v + c_1 * rand1 * (pBest - p) + c_2 * rand2 * (gBest - p) \qquad (7)$$

The modified value of a soil parameter is the sum of three terms which are the following.

- its present value multiplied by a weighting function
- the product of three parameters: a constant, the difference between its personal best value and the present value and a random number less than one
- the product of three parameters: a constant, the difference between its global best value and the present value and a random number less than one

While modifying the soil parameter values in matrix A, their personal best values can be extracted from the same

row of matrix B and the global best values as obtained from step 9 of the step-by-step procedure. In case the modified value of a parameter violates the upper limit or lower limit, its value is fixed as the limiting value it has violated. This check retains the trial values in the search space. c_1, c_2 and w can be assigned values as in any standard PSO algorithm. As this is a search for local optimum because of the nature of the problem it is ideal to choose a value of 0.4 for 'w'.

4 EXECUTION AND RESULTS

Table I and Table II give the Wenner resistivity measurement data. Table I gives electrode spacing for nine different set of measurement data and Table II gives the corresponding measured apparent resistivity values. Note that the first three sets of data are theoretically generated.

4.1 Test data

TABLE I
ELECTRODE SPACING

Set	Electrode spacing (m)							
1	1.0	2.0	3.0	4.0	5.0			
2	2.0	4.0	6.0	8.0	10.0			
3	2.0	4.0	6.0	8.0	10.0			
4	2.5	5.0	7.5	10.0	12.5	15.0		
5	1.0	1.5	2.5	3.0	5.0	10.0		
6	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
7	2.5	5.0	7.5	10.0	12.5	15.0	20.0	25.0
8	1.0	2.0	3.0	4.0				
9	1.0	2.0	4.0	10.0	20.0	40.0		

TABLE II
MEASURED VALUES OF APPARENT RESISTIVITYCORRESPONDING TO THE ELECTRODE SPACING GIVEN IN TABLE I

Set	Measured apparent resistivity (Ω-m)							
1	693.74	251.62	84.56	37.64	25.32			
2	123.33	189.99	258.93	320.27	374.13			
3	102.26	113.07	129.77	147.52	163.95			
4	320.00	245.00	182.00	162.00	168.00	152.00		
5	255.00	290.00	315.00	376.00	528.00	690.00		
6	58.71	61.79	58.10	61.00	73.79	78.00	79.13	78.19
7	451.60	366.70	250.20	180.00	144.20	120.00	115.50	96.50
8	156.40	113.10	95.20	65.30				
9	136.00	140.00	214.00	446.00	685.00	800.00		

Data courtesy: [1], [2]

4.2 Programming tips for PSO algorithm

Recommended values: c_1=2, c_2=2, w=0.4, initial value of k_1=2, final value of k_1=1.1, 'n' in (1)=2000
Convergence criterion: fitness >0. 999/iteration count >500

4.3 Programming tips *for Traditional algorithm*

The program is sensitive to the choice of α mentioned in step 4 of the step-by-step procedure. In this algorithm a value of 0.02 given for α
Convergence criterion: All the elements of C are less than tolerance, 0.0001.

4.4 Execution results

Execution results are shown in Table III. Execution is done using a 3.5 GHz, AMD FX -8320 Eight-Core processor with 4GB RAM.

TABLE III
EXECUTION RESULTS

Set	ρ1 (Ω-m)	ρ2 (Ω-m)	h m	Iteration count	Execution time (s)	F	Method
1	999.7140	19.9845	0.9999	500	72.95	0.00016	PSO
	999.7570	19.9903	0.9998	547	12.93	0.00040	Trad.
2	99.9987	998.5899	2.4995	500	49.12	0.00048	PSO
	100.0027	998.9895	2.4999	4495	92.27	0.00025	Trad.
3	100.0040	300.0793	5.0010	124	13.16	0.00015	PSO
	100.0043	299.9833	5.0004	1319	27.00	0.00006	Trad.
4	378.3809	144.9014	2.4971	500	72.94	0.1583	PSO
	365.8878	144.1014	2.8020	477	12.56	0.1847	Trad.
5	242.3719	983.6668	1.9818	500	62.58	0.1798	PSO
	244.2724	1004.70	2.0070	1916	45.96	0.1946	Trad.
6	58.1448	89.5127	1.2219	500	77.77	0.3624	PSO
	57.1118	93.0160	1.4608	610	16.36	0.3978	Trad.
7	481.0639	89.5661	4.5287	500	112.28	0.1887	PSO
	487.6667	91.5894	4.5063	486	17.04	0.2467	Trad.
8	173.4375	46.8050	1.3893	500	52.67	0.1409	PSO
	167.0466	44.9502	1.5793	421	8.27	0.1952	Trad.
9	133.0799	1033.10	3.0102	500	58.17	0.2862	PSO
	149.5676	1041.30	3.0801	2999	72.09	0.5457	Trad.

4.5 Comments on results

Comparison of results obtained by the two algorithms shows that PSO stands superior as far as simplicity and minimization are concerned. During execution it has been found that PSO doesn't need 500 iterations for optimal results, in most of the cases convergence has been obtained within 300 iterations. Moreover PSO algorithm eliminates all complex mathematical steps and the need of derivatives. Objective function values show that accuracy is more for PSO results.

4.6 Comparison of PSO results with GA solutions of Ioannis [3]

Table IV shows the comparison of PSO solutions with GA solutions of Ioannis [3]. In all the six cases PSO solutions

are found superior to GA solutions as far as accuracy is concerned. Also PSO works faster comparing with GA.

TABLE IV
COMPARISON OF PSO SOLUTION WITH GA SOLUTIONS OF IOANNIS [3]

Set of data	ρ1 (Ω-m)	ρ2 (Ω-m)	h m	F	Method
4	374.921	144.518	2.559	0.1600	**GA**
	378.3809	144.9014	2.4971	0.1583	**PSO**
5	243.419	986.960	2.000	0.1829	**GA**
	242.3719	983.6668	1.9818	0.1798	**PSO**
6	58.229	91.039	1.310	0.3635	**GA**
	58.1448	89.5127	1.2219	0.3624	**PSO**
7	499.827	89.847	4.409	0.2029	**GA**
	481.0639	89.5661	4.5287	0.1887	**PSO**
8	168.694	39.463	1.625	0.1512	**GA**
	173.4375	46.8050	1.3893	0.1409	**PSO**
9	128.645	1060.965	2.896	0.2928	**GA**
	133.0799	1033.10	3.0102	0.2862	**PSO**

5 CONCLUSION

Basically a PSO algorithm eliminates the need of solving a set of non-linear equations for optimal values of soil parameters. A normal PSO algorithm can yield such accurate results only with a large number of iterations. But, the accelerated algorithm yields highly accurate results with the least number of iterations in the PSO environment. The algorithm developed has been tested on nine different sets of data with three trials on each set of data. The strength of this algorithm lies in reducing the search space gradually about the most recent global best values of soil parameters. The comparative study of the PSO algorithm with the traditional algorithm proves the superiority of the PSO algorithm in respect of simplicity and accuracy.

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